

The complete radiative corrections to the gaugino and Higgsino masses in the Minimal Supersymmetric Model

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Abstract

We determine the radiative corrections to the masses of the gauginos and Higgsinos in the MSSM, including all sectors of the theory in a one-loop calculation in the on-mass-shell renormalization scheme. We find that a gluino which is massless at tree level receives a mass of between 0 and 3 GeV, primarily due to the top/stop contribution. This radiatively generated mass depends directly on the off-diagonal element of the squark mass matrix. In the case of a massive gluino, its mass receives typically large corrections, as large as 40% for a 125 GeV gluino. We find that the contributions to the neutralino and chargino mass corrections from the gauge/Higgs/gaugino/Higgsino sector are typically $\pm 1\%$. The lightest neutralino, which can receive corrections larger than 25%, receives $\mathcal{O}(5\%)$ corrections over most of the parameter space. We combine our results with the results of LEP and CDF searches to obtain the lower bounds on the neutralino and chargino masses at one-loop. We also demonstrate how the radiative corrections affect the presently excluded region of parameter space.

1 Introduction

In this paper we study the effects of radiative corrections on the gaugino and Higgsino masses in the minimal supersymmetric model. In a grand unified model the gaugino masses at the GUT scale are given by the common mass $m_{1/2}$. Using the one-loop renormalization group equations[1], the gaugino mass parameters M_1 , M_2 , and the gluino mass M_3 are given at a scale μ by

$$M_i(\mu^2) = \frac{\alpha_i(\mu^2)}{\alpha_{\text{GUT}}} m_{1/2} \quad (1)$$

which yields the relations $M_3 \simeq 3.5M_2$, and $M_1 \simeq 0.5M_2$. We assume these GUT relations in this paper. It is typical in analyses to assume that elements of the gaugino/Higgsino sector are lighter than the lightest elements of the squark/slepton sector, and this is required if the dark matter candidate is the lightest neutralino. In such a scenario, we might expect that members of the gaugino/Higgsino sector would be the first superpartners to be discovered at a particle collider. It is therefore of interest to determine the corrections that these particle masses receive. Until these particles are discovered, the corrections are useful in that they modify the region of parameter space which is ruled out by experiment. In the event that the gaugino and Higgsino particles are discovered these corrections would provide for a detailed check of the viability of the MSSM.

We divide the following discussion into four parts. In the next section we discuss the radiatively generated mass for a gluino which is massless at tree level. In Section 3 we discuss the corrections to the gluino mass in the case $m_{\tilde{g}} \gtrsim 100$ GeV. In section 4 we discuss the complete corrections to the neutralino and chargino masses, and the last section is reserved for the conclusions.

2 Light gluino mass

There is some controversy as to whether light gluinos ($m_{\tilde{g}} \lesssim 3$ GeV) have been ruled out by existing data[2]. The bounds from the non-observation of the decay mode $\Upsilon \rightarrow \gamma \eta_{\tilde{g}}$ [3] are model dependent[4, 5] ($\eta_{\tilde{g}}$ is the $\tilde{g}\tilde{g}$ pseudoscalar bound state). Ref.[5] concludes that these searches exclude $3 \text{ GeV} \lesssim m_{\eta_{\tilde{g}}} \lesssim 7 \text{ GeV}$, leaving the window $m_{\tilde{g}} \lesssim 1.5 \text{ GeV}$ open if $m_{\tilde{g}} = m_{\eta_{\tilde{g}}}/2$. Bounds determined from beam dump experiments[6] depend on the gluino lifetime and may not rule out very light gluinos if the squark masses are in the TeV range. Thus, it appears further experiments are needed to conclusively rule out the possibility of light gluinos, especially long lived gluinos with mass $m_{\tilde{g}} < 1 \text{ GeV}$ (see e.g. Ref.[7]). Interest in the possibility of light gluinos has been piqued recently in response to the observation that a light gluino could reconcile the apparent discrepancy between (central) values of $\alpha_s(M_Z^2)$ as determined from high and low energy experiments[4, 8].

In this section we examine the gluino mass generated entirely by one-loop radiative corrections in the event $M_3=0$. At one-loop, the solution to the RGE for the parameter M_3 , $M_3(\mu^2) = (\alpha_s(\mu^2)/\alpha_{\text{GUT}})m_{1/2}$, together with the GUT scale boundary condition $m_{1/2}=0$ would ensure $M_3=0$. Considering the two-loop RGE for the gluino mass[9], we see that if both $m_{1/2}$ and the A -terms vanish at the GUT scale, again we would find a massless gluino at

tree level. In any event, if the parameter $M_3(M_3^2)$ is small (\lesssim few GeV) then the correction given below can simply be added to it. The contribution to the gluino mass for $M_3 = 0$ due to the top/stop loop is

$$m_{\tilde{g}} = \frac{\alpha_s(m_t^2)}{4\pi} m_t \sin 2\theta_t \left(\frac{\ln r_1}{1 - r_1} - \frac{\ln r_2}{1 - r_2} \right), \quad (2)$$

where $r_{1,2} = m_t^2/m_{\tilde{t}_{1,2}}^2$ and θ_t is the angle which rotates the squarks from the left-right to the mass eigenstate basis. Changing the scale at which the strong coupling is evaluated from m_t (160 GeV) to $m_{\tilde{t}}$ (1 TeV) reduces the gluino mass by 14% (we set $\alpha_s(M_Z^2) = 0.12$). We sum over the six quarks, but only the top/stop contribution matters except at large values of $\tan \beta$, where the bottom/sbottom contribution can be non-negligible. We note here that our result for the gluino mass, as well as the photino mass, is larger by a factor of 2 relative to those of Refs. [4, 10]. We believe that our result is correct since the cancellation of infinities in the gluino and neutralino masses depends on it.

For a given flavor, the squark masses and rotating angle θ are determined from the squark mass matrix. For up type squarks, for example, we have

$$\mathbf{M}_{\text{sq}}^2 = \begin{pmatrix} a_u & c_u \\ c_u & b_u \end{pmatrix} \quad (3)$$

where

$$a_u = M_{\tilde{Q}}^2 + M_Z^2 \cos 2\beta \left(1/2 - 2/3 s_w^2 \right) + m_u^2 \quad (4)$$

$$c_u = m_u (A_u - \mu \cot \beta) \quad (5)$$

$$b_u = M_{\tilde{U}}^2 + 2/3 M_Z^2 \cos 2\beta s_w^2 + m_u^2 \quad (6)$$

We prefer to present our results in terms of the underlying parameters $M_{\tilde{Q}}$, $M_{\tilde{U}}$, μ , and A . We discuss the results for a common A -term, $A = A_t = A_b$, and a common right-handed squark mass $M_{\tilde{U}} = M_{\tilde{D}}$. We set the top quark mass to 160 GeV. In Fig.1(a) we show gluino mass contours in the $M_{\tilde{Q}}$, μ plane, with $\tan \beta = 1.5$, $M_{\tilde{U}} = M_{\tilde{D}} = 200$ GeV, and $A=0$. We consider the range $|\mu| < 800$ GeV, and $100 < M_{\tilde{Q}} < 1000$ GeV. We note that the gluino mass is at most about 3 GeV. It is equal to 0 near $\mu = A \tan \beta$, since in this case there is no mixing between left and right stop squarks. The mass increases as we move away from the no-mixing line until we reach the boundary of the parameter space, defined by the requirement that the squared squark masses are positive. The \times 's on Figs.1(a,c,d) mark unphysical regions of parameter space. As we increase $M_{\tilde{U}}, M_{\tilde{D}}$ the allowed parameter space increases and the

Figure 1: Contours of the radiatively generated gluino mass for a gluino which is massless at tree level, in the $M_{\tilde{Q}}$, μ plane. In Figs.(a,b,c) the contours are symmetric in $\mu \rightarrow -\mu$. The contours are labeled in GeV. The \times 's mark unphysical regions of parameter space.

bound on the gluino mass generally decreases. For example, in Fig.1(b) $M_{\tilde{U}}$ and $M_{\tilde{D}}$ are increased to 1 TeV, and the maximum gluino mass is about 700 MeV. The dependence of the contour plots in Figs.1(a,b) on $\tan\beta$ and A is easily understood, since for small $\tan\beta$ the gluino mass depends directly on the parameters A , μ , and $\tan\beta$ through the combination $A - \mu \cot\beta$. Therefore changing A or $\tan\beta$ amounts to simply a shift or rescaling of the y-axis of these figures. For large $\tan\beta$ the dependence is more complicated, since the bottom-sbottom sector becomes non-negligible, and the parameter space is more severely constrained. In Fig.1(c) we show the gluino mass contours for $\tan\beta = 30$, $M_{\tilde{U}} = M_{\tilde{D}} = 200$ GeV, and $A = 0$. The gluino mass in this case varies from 0 at $\mu=0$ to about 400 MeV. Again, the bound is reduced if we increase M_U, M_D . For $M_U = M_D = 1$ TeV and $\tan\beta = 30$ the gluino mass is at most 120 MeV. As we vary A when $\tan\beta$ is large the pattern of the gluino mass contours in the $M_{\tilde{Q}}, \mu$ plane changes dramatically, as can be seen in Fig.1(d), where A is increased to 300 GeV. Finally, we note that if we instead examine contours in the $M_{\tilde{U}}, \mu$ plane keeping $M_{\tilde{Q}}$ constant, we get essentially the same results. This is due to the fact that in a_u, b_u defined above, $M_{\tilde{Q}}$ and $M_{\tilde{U}}$ dominate the contribution of the D-terms which are proportional to M_Z^2 . This is true especially at $\tan\beta \simeq 1$ but it persists for larger values of $\tan\beta$ as well.

3 Heavy gluino mass

In this section we consider radiative corrections to the gluino mass in the case $M_3 \neq 0$. Naturally the gluino mass limits referred to in this section assume that light gluinos are excluded. The CDF collaboration obtained the bound $m_{\tilde{g}} > 141$ GeV under the unrealistic assumption that the branching ratio $B(\tilde{g} \rightarrow \chi_1^0 q \bar{q}) = 1$ [11]. They also included the effects of cascade decays[12] in their analysis and, for a particular choice of parameters, obtained a bound $m_{\tilde{g}} > 95$ GeV [11]. If the unification condition for the gaugino masses is assumed, combining the LEP limits on chargino and neutralino masses[13] with the gluino cascade analysis further constrains the gluino mass bound (and the bound then becomes μ and $\tan\beta$ dependent). Such an analysis was carried out in Ref.[14], where, for a typical choice of parameters, they obtained $m_{\tilde{g}} > 135$. Hidaka [15] performed a similar analysis, obtaining the μ and $\tan\beta$ independent bound $m_{\tilde{g}} > 132$. However, the starting point for both of these last two reference's analysis was $m_{\tilde{g}} > 150$ GeV with $B(g \rightarrow \chi_1^0 q \bar{q}) = 1$, a preliminary CDF result. Hence, these bounds should be relaxed accordingly. A reasonable lower bound on the gluino mass is $m_{\tilde{g}} > 125$ GeV [16].

The one-loop corrected gluino mass is given by

$$\begin{aligned}
m_{\tilde{g}} = & M_3(\mu^2) + \frac{3\alpha_s}{4\pi} M_3 \left(5 - 3 \ln \left(\frac{M_3^2}{\mu^2} \right) \right) \\
& - \sum_{q=u,\dots,t} \frac{\alpha_s}{4\pi} M_3 \operatorname{Re} \left[\hat{B}_1(M_3^2, m_q^2, m_{\tilde{q}_1}^2) + \hat{B}_1(M_3^2, m_q^2, m_{\tilde{q}_2}^2) \right] \\
& + \sum_{q=t,b} \frac{\alpha_s}{4\pi} m_q \sin 2\theta_q \operatorname{Re} \left[B_0(M_3^2, m_q^2, m_{\tilde{q}_1}^2) - B_0(M_3^2, m_q^2, m_{\tilde{q}_2}^2) \right],
\end{aligned} \tag{7}$$

where we evaluate α_s at the scale M_3 . We take the tree level gluino mass to be $M_3(M_3^2)$. The functions B_0 and B_1 can be found in Ref.[17], except that here we adopt the metric $(+, -, -, -)$. The hatted B_1 's refer to the function B_1 with the infinite part (proportional to $1/\epsilon + \ln 4\pi - \gamma_E$) subtracted. We list in Eq.(7) the gluon contribution, followed by the quark/squark contributions. In the first sum over quarks we include all six flavors, while in the second we indicate that the sum is relevant only for t and b quarks. We note that the one-loop level μ dependence of the parameter $M_3(\mu^2)$ in Eq.(7) cancels against the μ dependence of the one-loop correction.

Given a measurement of the physical gluino mass, we are interested in determining the value of the underlying parameter M_3 , which will then yield a prediction for the other gaugino mass parameters M_1 and M_2 via eq.(1). To this end we plot in Fig.2(a) M_3 vs. $m_{\tilde{g}}$, for $m_t=160$ GeV. For the remainder of this paper, we will refer to a common squark mass parameter m_{sq} , defined by $m_{sq} = M_{\tilde{Q}} = M_{\tilde{U}} = M_{\tilde{D}} = 2A$, where A is the common A -term. (In the plots of Figs.2 we set $\tan\beta=1.5$ and $\mu = -200$ GeV, but the results are insensitive to

Figure 2: (a) The tree level gluino mass M_3 vs. the one-loop gluino mass $m_{\tilde{g}}$ for $m_t = 160$ GeV and two values of the squark mass, $m_{sq} = 160$ GeV and $m_{sq} = 1000$ GeV. The dotted line indicates the tree level relation $m_{\tilde{g}} = M_3$. (b) The relative difference between $m_{\tilde{g}}$ and M_3 , vs. M_3 for three values of the squark mass.

the values of these parameters). We see that a measured mass $m_{\tilde{g}} = 100$ GeV would imply $M_3=69$ GeV (84 GeV) for $m_{sq} = 1000$ GeV (160 GeV). This is to be compared with the tree level relation $m_{\tilde{g}} = M_3$. This amounts to a 15% to 40% correction to this relation for gluino masses at the present limit of 125 GeV. In a grand unified context the gluino mass would typically be taken to be the parameter M_3 evaluated at the scale M_3 . We show the error in neglecting the low energy threshold corrections in this case in Fig.2(b), where we plot the relative correction to the gluino mass vs. M_3 for three values of the squark mass parameter m_{sq} . The left endpoints of the curves in Fig.2(b) correspond to $m_{\tilde{g}} = 100$ GeV. For values of M_3 less than the quark/squark thresholds the corrections are generally appreciable. However, once M_3 is larger than the quark/squark thresholds the relative correction to the tree level mass is small ($\sim 1\%$) and asymptotically approaches the value $(3\alpha_s)/(4\pi)$.

4 Chargino and neutralino masses

In the MSSM the superpartners of the neutral Higgs bosons and $SU(2) \times U(1)$ gauge bosons mix to form the mass eigenstates (in order of increasing mass) χ_1^0 , χ_2^0 , χ_3^0 , and χ_4^0 . Similarly, the charginos are the mass eigenstates of the charged gauginos and Higgsinos, and are denoted χ_1^\pm and χ_2^\pm (see Ref.[18]). A lower bound on the lightest neutralino mass in the MSSM was first obtained using combined LEP and UA2/CDF data in Ref.[19], and more recently lower bounds on all of the neutralino and chargino masses were given in Ref.[15]. The corrections to the neutralino and chargino masses were studied in Ref.[20] including only the

top/stop sector contributions, and in Ref.[21], including the lepton/slepton and quark/squark contributions. In this section we present the the complete corrections to the chargino and neutralino masses, including the gauge/Higgs/gaugino/Higgsino sector, thus completing the calculation of Ref.[21]. In Ref.[21] we found the corrections due to matter loops were commonly of order 5%. As the corrections due to the gauge/Higgs/gaugino/Higgsino sector are expected to be of this same order, they must be included in order to obtain a reliable result.

For consistency we use the same formalism as in Ref.[21] which we shall not repeat here. We point out one inconsequential modification, which we adopt for technical reasons. Specifically, when computing the masses, defined as the poles of the relevant two-point Greens functions, we need to determine counterterms for the various input parameters that enter the calculation. One of these counterterms is $\delta\beta$, where $\tan\beta$ is the ratio of the the two vevs of the Higgs fields H_1 and H_2 . In our renormalization scheme we take $\delta\beta$ to be purely infinite, i.e. proportional to $1/\epsilon$ in dimensional regularization. Since $\delta\beta$ is purely infinite, it will not contribute to the final finite result. Yet, it is important to have an expression for it in order to check the crucial cancellation of infinities, which must occur in a renormalizable theory. In Ref.[21] we determined $\delta\beta$ from the Higgs sector. In the present work we determine it from the chargino-neutralino sector. In particular, we demand that the the chargino masses, as well as the trace of the eigenvalues and the squares of the eigenvalues of the neutralino mass matrix, are finite. This determines the counterterms δM_1 , δM_2 , $\delta\mu$ and $\delta\beta$. Notice that even after imposing finiteness of these quantities it is still nontrivial to check that all of the individual neutralino masses are finite. In addition, we explicitly checked that we get the same value for $\delta\beta$ whether we calculate it from the Higgs sector or the chargino-neutralino sector.

The self-energies of the gauge bosons, which enter in the computation of the counterterms, can be found in Ref.[22]¹. The physical chargino and neutralino masses are given by [23]

$$M_{\chi_i^+} = M_{\chi_{ir}^+} + \delta M_{\chi_i^+} - \text{Re} \left(\Sigma_{1ii}^+ (M_{\chi_i^+}^2) + M_{\chi_i^+} \Sigma_{\gamma ii}^+ (M_{\chi_i^+}^2) \right) \quad (8)$$

$$M_{\chi_\alpha^0} = M_{\chi_{\alpha r}^0} + \delta M_{\chi_\alpha^0} - \text{Re} \left(\Sigma_{1\alpha\alpha}^0 (M_{\chi_\alpha^0}^2) + M_{\chi_\alpha^0} \Sigma_{\gamma\alpha\alpha}^0 (M_{\chi_\alpha^0}^2) \right) \quad (9)$$

where the Σ 's are form factors of the one-loop fermion self-energy

$$\Sigma_{ij} = \Sigma_{1ij} + \Sigma_{5ij} \gamma_5 + \Sigma_{\gamma ij} \not{p} + \Sigma_{5\gamma ij} \not{p} \gamma_5, \quad (10)$$

and the $M_{\chi_r} + \delta M_\chi$ are the masses obtained by replacing each bare parameter x_{i_b} in the tree level neutralino and chargino mass formulas by the renormalized parameter plus the corresponding counterterm, $x_{i_b} \rightarrow x_{i_r} + \delta x_i$. (Here the x_i refer to the parameters M_W , M_Z , M_1 , M_2 , μ , and $\tan\beta$; see Ref.[21] for details.) The gauge/Higgs/gaugino/Higgsino contributions to the relevant chargino and neutralino self-energies are given in the Appendix.

¹The definition of the gauge boson self-energies given in Ref.[21] differ by a sign relative to those of Ref.[22]. Similarly, our definition of the function B_1 differs by a sign from Ref.[22]. We point out two typographical errors in the formulas for the gauge boson self-energies given in Ref.[22]. First, in the formula for Π_Z^T , Eq.(A.15), s_W^2 in the term $-2g_2^2 s_W^2 M_Z^2 b_0 (M_W^2, M_W^2)$ should be replaced by s_W^4 . Second, in the equation for Π_W^T , Eq.(A.20), the g_2^2 in the term $-\frac{1}{4}g_2^2 \sum_{i=1}^2 (C_R^i)^2 b_0 (M_W^2, m_{H_i^0}^2)$ should be g_2^4 .

We set the top mass to 160 GeV and the CP -odd Higgs boson mass m_A to $m_{sq}/2$ in the following. The results depend mildly on the scale of the Higgs boson sector, especially for any $m_A > M_Z$. The tree level masses are functions of the underlying parameters M_1 , M_2 , μ and $\tan\beta$. In our renormalization scheme these parameters are identified with the \overline{DR} -running parameters $M_1(Q^2)$, etc., and hence they have a renormalization scale dependence. While the Q dependence cancels in the result for the physical masses, the corrections (ΔM_χ or $\Delta M_\chi/M_\chi$) do depend on the scales chosen for the evaluation of the tree level masses. We

Figure 3: Contours of constant neutralino mass in the μ , M_2 plane with $\tan\beta = \sqrt{2}$ at tree level (dashed lines) and one-loop level (solid lines). The light (dark + light) shading indicates the region of parameter space presently ruled out by LEP, including (not including) the one-loop corrections. Additionally the region below the dotted (dash-dotted) line is the tree (one-loop) level region ruled out by CDF gluino searches. The contours are labeled in GeV.

Figure 4: Same as Fig.3, with $\tan\beta = 40$. The mass contours are shown for (a) the lightest neutralino, (b) the lighter chargino, and (c) the heavier chargino. In this case the gluino mass bound gives no additional constraint.

choose to evaluate the tree level masses with the parameters $M_1(M_1^2)$, $M_2(M_2^2)$, $\mu(\mu^2)$, and $\tan\beta(M_Z^2)$.

In Figs.3(a-c) we plot the mass contours of χ_1^0 , χ_2^0 and χ_3^0 in the μ , M_2 plane at tree (dashed lines) and one-loop (solid lines) level, for $\tan\beta = \sqrt{2}$ and $m_{sq} = 1$ TeV. In Figs.4(a-c) we show the mass contours of χ_1^0 , χ_1^\pm , and χ_2^\pm , for $\tan\beta = 40$. The shaded regions on these figures indicate the excluded regions of parameter space due to negative searches and line-shape measurements at LEP[13]. Because the corrections are in general positive, there is a smaller region of parameter space ruled out at one-loop level. The light shaded regions shown in Figs.3 and 4 are ruled out at one-loop level, while the adjacent strips of dark shading indicate the additional region which is ruled out if the chargino and neutralino masses are considered at tree level. The boundary of the excluded region is determined largely from the bound $M_{\chi_1^\pm} > 47$ GeV [13]. The region below the dotted (dash-dotted) horizontal line in Figs.3 is excluded at tree level (one-loop level) by considering the non-observation of gluinos by CDF and assuming the GUT relation Eq.(1). The horizontal lines on the plots correspond to $m_{\tilde{g}} > 125$ GeV, which translates by Eqs.(1,7) into $M_2 > 36$ GeV (25 GeV) at tree level (one-loop level). The bound $m_{\tilde{g}} > 125$ GeV provides no additional excluded region beyond the region that is already excluded by LEP for $\tan\beta \gtrsim 2$.

We note that for a given $\tan\beta$ the results for the lightest chargino and neutralino are qualitatively similar, and the same is true for the heaviest chargino and neutralino. For $\tan\beta = \sqrt{2}$ and $\mu = -200$ GeV a measured χ_1^0 mass of 100 GeV would imply $M_2 = 192$ GeV if the tree level relation is used, while taking into account radiative corrections we obtain

$M_2 = 185$ GeV. Generically, the corrections ΔM_χ are below 10 GeV (12 GeV) in the region $M_2 \lesssim 120$ GeV, $|\mu| \lesssim 200$ GeV at $\tan\beta=\sqrt{2}$ ($\tan\beta=40$) and they increase as we move away from the excluded region. For the next-to-lightest neutralino and lightest chargino with $|\mu| \simeq 350$ GeV and $M_2 \simeq 250$ GeV we can obtain a correction ΔM_χ greater than 14 GeV.

Figure 5: Contours of the percentage difference between the tree and one-loop level values of the lightest neutralino mass in the μ , M_2 plane with $\tan\beta = 2$. The solid lines (with large labels) indicate the results for $m_{sq} = 1$ TeV, while the $m_{sq} = 180$ GeV results are indicated with dotted lines (small labels). The shading indicates the one-loop level excluded region with $m_{sq} = 1$ TeV.

We illustrate that the corrections depend strongly on the squark mass in Fig.5, where we show a contour plot of the percentage correction for the lightest neutralino mass in the μ , M_2 plane for $\tan\beta = 2$ and $m_{sq} = 1$ TeV (solid lines), as well as for $m_{sq} = 180$ GeV (dotted lines). Also shown is the excluded region at one-loop with $m_{sq} = 1$ TeV. We see that in most of the parameter space the correction is $3 \sim 8\%$ for $m_{sq}=1$ TeV, and $-1 \sim 4\%$ for $m_{sq} = 180$ GeV. For $m_{sq} = 1$ TeV the corrections are significantly larger for smaller $\tan\beta$ or larger m_t . The 12% contour line shown in Fig.5 increases to 23% if $\tan\beta = 1$.

Generally, the relative corrections are smaller for the heavier particles. For example, with $\tan\beta = \sqrt{2}$ and $m_{sq}=1$ TeV we obtain, in the region of parameter space shown in Fig.5, a correction between 5% and 15% for χ_1^+ , a 3-7% correction for χ_2^0 , and for χ_3^0 , χ_4^0 and χ_2^+ a correction of 3-4%.

Figure 6: The relative difference (in percent) between the tree level and one-loop level values of the chargino and neutralino masses vs. $\tan\beta$ for $m_t = 160$ GeV and $m_{sq} = 1$ TeV. (a) $\mu = -400$ GeV, $M_2 = 100$ GeV. (b) $\mu = -100$ GeV, $M_2 = 400$ GeV. The chargino curves are indicated with dot-dashed lines.

We find that the corrections can depend strongly on $\tan\beta$. In Figs.6(a,b) we plot the relative correction to the neutralino and chargino masses vs. $\tan\beta$ for two different choices of M_2 and μ . We note that for a given choice of the parameters M_2 and μ typically the lightest chargino and the two lightest neutralinos will be predominantly Higgsinos, with the heavier particles being predominantly gauginos, or vice versa. The relative correction is fairly independent of $\tan\beta$ for the gauginos. However, when the particle in question is mainly a Higgsino, as we increase $\tan\beta$ from 1 to 2 the correction decreases as the top Yukawa coupling decreases. For $2 \lesssim \tan\beta \lesssim 10$ the Higgsino corrections are fairly constant. Increasing $\tan\beta$ further, the bottom Yukawa becomes important and $\Delta M_\chi/M_\chi$ increases again. The relative corrections at $\tan\beta=1$ and $\tan\beta=30$ are about equal.

The gauge/Higgs/gaugino/Higgsino contributions to the corrections are comparable to those of the matter sector in certain regions of parameter space. For example, for $\mu \simeq$

-400 GeV, $M_2 \simeq 400$ GeV, $\tan\beta = 1.5$ and $m_{sq} = 1$ TeV, the correction due to the matter sector is 5 GeV for χ_4^0 and χ_2^+ , while the correction from the gauge/Higgs sector is about 7 GeV. Similarly, for χ_2^0 and χ_1^+ in this region the matter correction is 7 GeV, while the gauge correction is 5 to 6 GeV. The gauge/Higgs relative corrections are small; in the experimentally allowed part of the region $|\mu| < 500$ GeV, $M_2 < 400$ GeV, we find the corrections are usually in the range -1 to 1% , and they are always less than 3% .

To conclude this section we list in Table 1 the lower bounds on the neutralino and chargino masses, independent of M_2 , μ , and $\tan\beta$ (we require $\tan\beta \geq 1$). In the first

	χ_1^0	χ_2^0	χ_3^0	χ_4^0	χ_1^+	χ_2^+
M_χ^{\min} (GeV)	16	44	70	108	47	99
$\frac{\Delta M_\chi^{\min}}{M_\chi^{\min}}$ (%) $m_t=160, m_{sq}=1$	-18	-1.2	0.3	0.8	0	1.0
$\frac{\Delta M_\chi^{\min}}{M_\chi^{\min}}$ (%) $m_t=200, m_{sq}=1.6$	-20	-3.5	-0.1	-1.0	0	-0.8

Table 1: Lower bounds on the neutralino and chargino masses. Row 1 lists the one-loop lower bounds in GeV with $m_t = 160$ GeV and $m_{sq} = 1$ TeV. Row 2 shows the percent difference between the lower bounds as determined at tree and one-loop level. Row 3 is as row 2, with $m_t = 200$ GeV and $m_{sq} = 1.6$ TeV.

row, we show the lower bounds for the masses of the various particles at one-loop level, with $m_t = 160$ GeV and $m_{sq}=1$ TeV. We determine these limits by finding the minimum χ mass in the experimentally allowed region of parameter space, where both the masses and the boundary of the excluded region include the one-loop mass corrections. In particular we have the limits from the CDF cascade analysis $m_{\tilde{g}} > 125$ GeV [11, 15, 16], and from LEP searches $M_{\chi_1^+} > 47$ GeV [13]. We note that, excepting the lightest neutralino, these are essentially the same limits found at tree level in Ref.[15]. In the second row of Table 1 we list the relative difference (in percent) between our tree level determination of the lower bounds and the one-loop results. In particular, we find that the minimum χ_1^0 mass at tree level is 19.6 GeV. Excepting χ_1^0 , the difference between the tree and one-loop determinations are all about 1% or less. These small differences are the result of cancellations. Both the boundary of the excluded region and the mass contours of all the particles shift in the same direction, so that the net effect is small. This cancellation is illustrated in Fig.4(a) of Ref.[21]. By way of contrast, for χ_2^0 (χ_3^0), the masses at tree and one-loop level differ by 10% (5%) for fixed μ , M_2 and $\tan\beta$ in the neighborhood of the minimum. Because the contours all shift relative to each other the cancellation is independent of the parameters which affect the overall size of the corrections. This is illustrated in the third row of Table 1, where we show the relative difference between the tree level and one-loop level lower bounds, setting $m_t = 200$ GeV and $m_{sq} = 1.6$ TeV. Again, excepting χ_1^0 , the differences are small.

The cancellation of electroweak corrections is not apparent for the lower bound deduced for the mass of the lightest neutralino because it finds its minimum value at a point in parameter space at which the gluino mass takes its lowest value. Thus, because the gluino mass bound receives strong corrections as discussed in Section 3 it is clear that the cancellations

cannot occur for χ_1^0 . However, there is some cancellation taking place. For a gluino mass correction of 30%, the corresponding 30% shift in M_2 (and M_1) changes the tree level χ_1^0 mass by about 27%. The one-loop correction to the χ_1^0 mass partially compensates for this correction, and the net effect is about a 20% shift.

The chargino and neutralino mass limits are not likely to increase much in the near future. Excepting the lightest neutralino, all of the particles find their minimum values at points in parameter space essentially at the kinematic limit of LEP. Additionally, the points are far from the gluino mass bound. The χ_1^0 mass limit increases proportionally to the gluino mass bound as $m_{\chi_1^0}^{\min} \simeq m_{\tilde{g}}^{\min}/7.8$ for gluino mass limits up to 155 GeV. The χ_2^0 mass limit remains unaffected for gluino mass bounds up to 180 GeV, the χ_4^0 and χ_2^+ limits are unaffected for gluino mass bounds up to 220 GeV, and the χ_3^0 limit is independent of the gluino mass limit up to $m_{\tilde{g}}^{\min} = 400$ GeV.

5 Conclusions

We have computed all of the one-loop radiative corrections to the gaugino and Higgsino masses in the MSSM. We performed the diagrammatic calculation in the on-mass-shell renormalization scheme.

We first considered the radiatively generated mass for a gluino which is massless at tree level. This mass falls generally in the “light gluino window”, $m_{\tilde{g}} \lesssim 3$ GeV. There is by no means a consensus in the literature as to whether gluinos in this mass range have already been excluded by current experimental data. If such a light gluino were discovered, it would be compelling evidence for the condition $m_{1/2} = 0$ at the GUT scale, and it would greatly constrain the parameters in the top/stop sector of the MSSM.

Next, we discussed the corrections to the mass of a massive gluino. We found large $\mathcal{O}(20\%)$ corrections for values of the gluino mass below the quark/squark thresholds, and small $\mathcal{O}(1\%)$ corrections for gluino masses above the quark/squark thresholds. These corrections must be considered when relating the gluino mass to the mass parameters M_1 and M_2 which enter into the evaluation of the chargino and neutralino masses.

Lastly, we discussed the results for the corrections to the neutralino and chargino masses. While the corrections from the gauge/Higgs sector can be as large as those from the matter sector, the gauge/Higgs sector corrections are never as large as 3%. The complete corrections are typically 3-8%. These corrections are relevant when determining the values of the underlying parameters from a set of physical χ masses. The underlying parameters can then be used to constrain or make predictions for other sectors of the theory. For example, μ enters into the Higgs boson and heavy quark/squark sectors, and $\tan\beta$ enters into the Higgs, fermion/sfermion and quark/squark sectors. Alternatively, given a set of underlying parameters μ , $m_{1/2}$, etc., at some high (e.g. GUT) scale, the parameters can be determined at a low scale via the renormalization group equations and the corrections presented here can then determine the physical spectrum of gaugino and Higgsino masses. Of course the physical spectrum is independent of the scale at which the RGE evolution is terminated, up to higher orders in perturbation theory.

We showed how the presently excluded region of parameter space is affected when the

mass corrections are considered, and we found the one-loop lower bounds on the neutralino and chargino masses. Excepting the lightest neutralino, the difference between the tree and one-loop determinations of the lower bounds were found to be small ($\sim 1\%$), due to cancellations. The χ_1^0 lower bound receives 20% corrections.

Appendix

Below we list the relevant one loop self-energies for the charginos and neutralinos, including only the gauge/Higgs particles and their partners in loop. The matter loop contributions can be found in Ref. [21]. The functions B_0 , B_1 are given in Ref.[17], except that here we adopt the Minkowski metric $(+, -, -, -)$. The various A and B couplings in the following expressions are the vector and axial vector (scalar and axial scalar) couplings of the charginos and neutralinos to the gauge bosons (Higgs particles). They can be found in Refs.[18, 24]. The indices i, j refer to charginos while the indices α, β refer to neutralinos. The H_n^0 , $n = 1..4$ denote the neutral Higgs bosons, H, h, A, G^0 , while H_m^+ , $m = 1, 2$, denote the charged Higgs bosons, H^+ , and G^+ . The calculation was performed in the Feynman gauge, in which $M_{G^0} = M_Z$ and $M_{G^+} = M_W$. Dimensional reduction[25] was used in regularizing the theory.

We have for the charginos

$$\begin{aligned}
(4\pi)^2 \Sigma_{1ii}^+(p^2) = & \sum_{j=1}^2 \sum_{n=1}^4 M_{\chi_j^+} \left((A_{ijn})^2 - (B_{ijn})^2 \right) B_0(p^2, M_{\chi_j^+}^2, M_{H_n^0}^2) \\
& + \sum_{\beta=1}^4 \sum_{m=1}^2 M_{\chi_\beta^0} \left(|A_{\beta im}|^2 - |B_{\beta im}|^2 \right) B_0(p^2, M_{\chi_\beta^0}^2, M_{H_m^+}^2) \\
& - 4 \sum_{j=1}^2 M_{\chi_j^+} \left((A_{ijZ})^2 - (B_{ijZ})^2 \right) B_0(p^2, M_{\chi_j^+}^2, M_Z^2) \\
& - 4e^2 M_{\chi_i^+} B_0(p^2, M_{\chi_i^+}^2, 0) \\
& - 4 \sum_{\beta=1}^4 M_{\chi_\beta^0} \left(|A_{\beta iZ}|^2 - |B_{\beta iZ}|^2 \right) B_0(p^2, M_{\chi_\beta^0}^2, M_W^2)
\end{aligned} \tag{A1}$$

$$\begin{aligned}
(4\pi)^2 \Sigma_{\gamma ii}^+(p^2) = & \sum_{j=1}^2 \sum_{n=1}^4 \left((A_{ijn})^2 + (B_{ijn})^2 \right) B_1(p^2, M_{\chi_j^+}^2, M_{H_n^0}^2) \\
& + \sum_{\beta=1}^4 \sum_{m=1}^2 \left(|A_{\beta im}|^2 + |B_{\beta im}|^2 \right) B_1(p^2, M_{\chi_\beta^0}^2, M_{H_m^+}^2) \\
& + 2 \sum_{j=1}^2 \left((A_{ijZ})^2 + (B_{ijZ})^2 \right) B_1(p^2, M_{\chi_j^+}^2, M_Z^2) \\
& + 2e^2 B_1(p^2, M_{\chi_i^+}^2, 0) \\
& + 2 \sum_{\beta=1}^4 \left(|A_{\beta iW}|^2 + |B_{\beta iW}|^2 \right) B_1(p^2, M_{\chi_\beta^0}^2, M_W^2)
\end{aligned} \tag{A2}$$

The corresponding expressions for the neutralinos are

$$\begin{aligned}
(4\pi)^2 \Sigma_{1\alpha\alpha}^0(p^2) = & \sum_{\beta=1}^4 \sum_{n=1}^4 M_{\chi_\beta^0} \left(|A_{\alpha\beta n}|^2 - |B_{\alpha\beta n}|^2 \right) B_0(p^2, M_{\chi_\beta^0}^2, M_{H_n^0}^2) \\
& + 2 \sum_{j=1}^2 \sum_{m=1}^2 M_{\chi_j^+} \left(|A_{\alpha jm}|^2 - |B_{\alpha jm}|^2 \right) B_0(p^2, M_{\chi_j^+}^2, M_{H_m^+}^2) \quad (\text{A3}) \\
& - 4 \sum_{\beta=1}^4 M_{\chi_\beta^0} \left(|A_{\alpha\beta Z}|^2 - |B_{\alpha\beta Z}|^2 \right) B_0(p^2, M_{\chi_\beta^0}^2, M_Z^2) \\
& - 8 \sum_{j=1}^2 M_{\chi_j^+} \left(|A_{\alpha j W}|^2 - |B_{\alpha j W}|^2 \right) B_0(p^2, M_{\chi_j^+}^2, M_W^2)
\end{aligned}$$

$$\begin{aligned}
(4\pi)^2 \Sigma_{\gamma\alpha\alpha}^0(p^2) = & \sum_{\beta=4}^2 \sum_{n=1}^4 \left(|A_{\alpha\beta n}|^2 + |B_{\alpha\beta n}|^2 \right) B_1(p^2, M_{\chi_\beta^0}^2, M_{H_n^0}^2) \\
& + 2 \sum_{j=2}^4 \sum_{m=1}^2 \left(|A_{\alpha jm}|^2 + |B_{\alpha jm}|^2 \right) B_1(p^2, M_{\chi_j^+}^2, M_{H_m^+}^2) \quad (\text{A4}) \\
& + 2 \sum_{\beta=1}^4 \left(|A_{\alpha\beta Z}|^2 + |B_{\alpha\beta Z}|^2 \right) B_1(p^2, M_{\chi_\beta^0}^2, M_Z^2) \\
& + 4 \sum_{j=1}^2 \left(|A_{\alpha j W}|^2 + |B_{\alpha j W}|^2 \right) B_1(p^2, M_{\chi_j^+}^2, M_W^2)
\end{aligned}$$

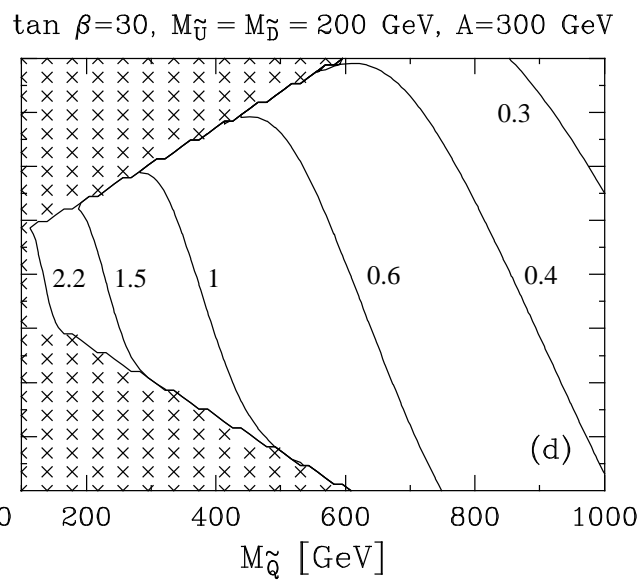
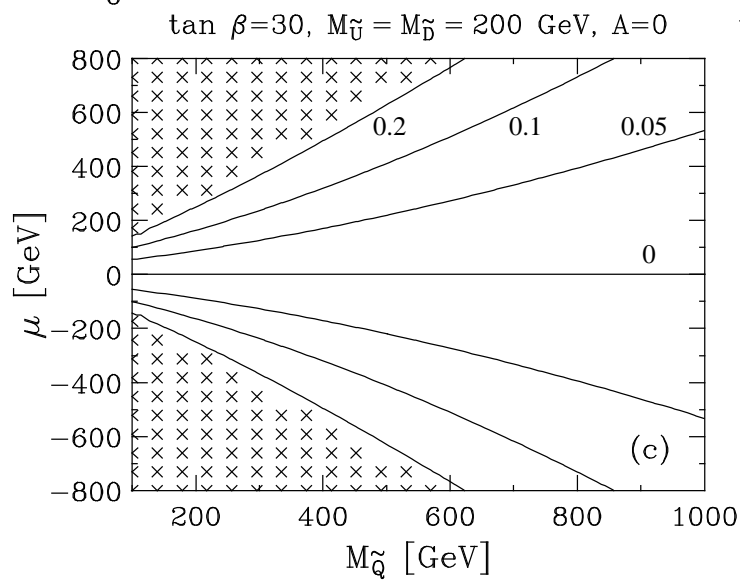
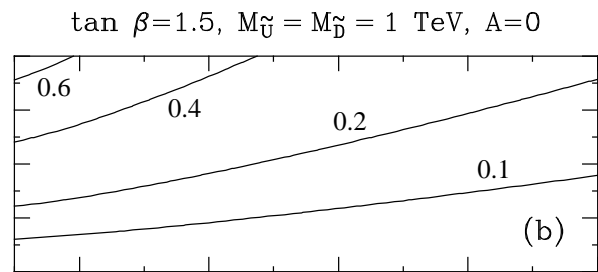
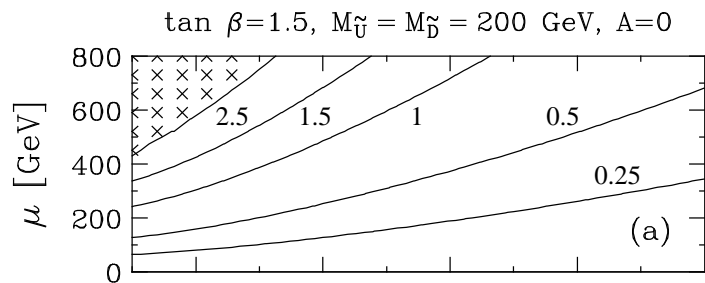
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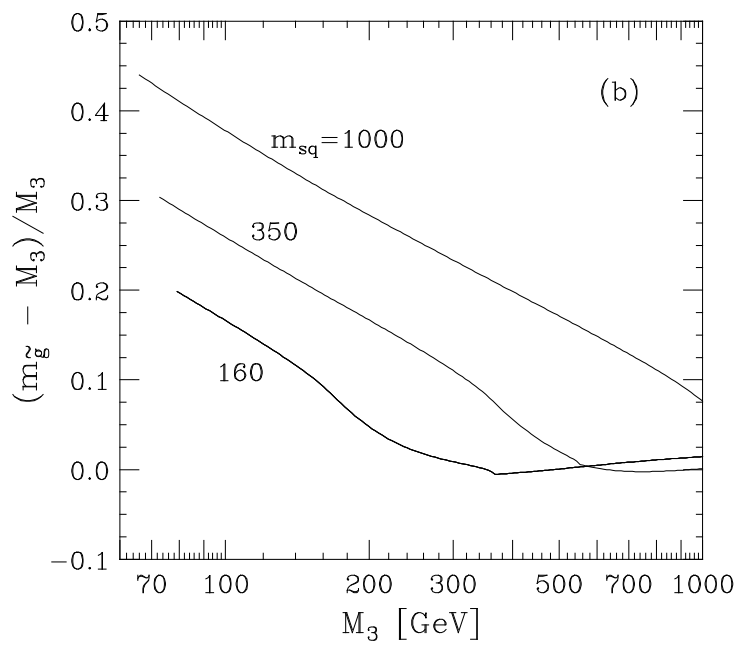
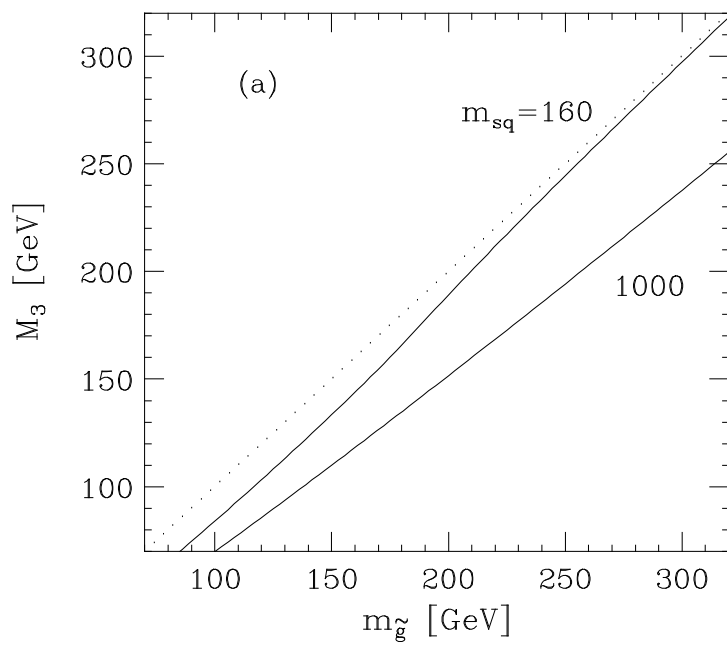
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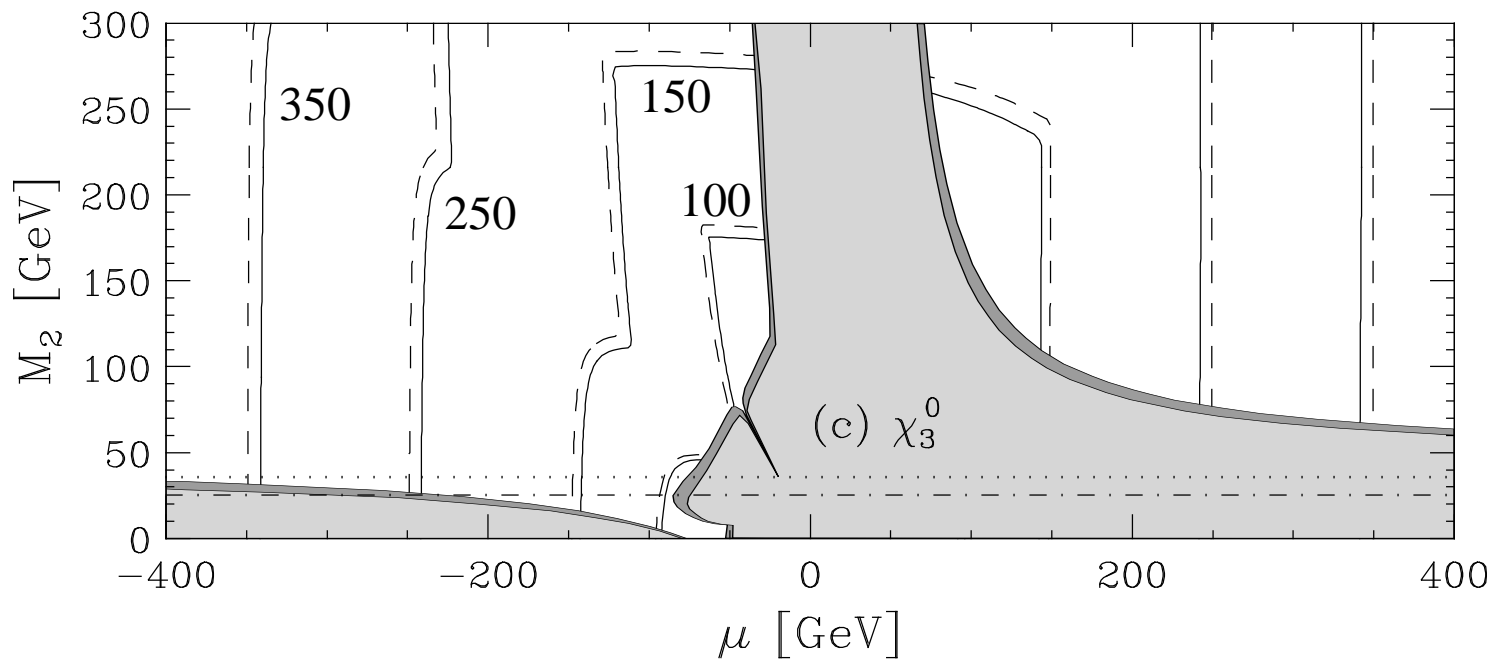
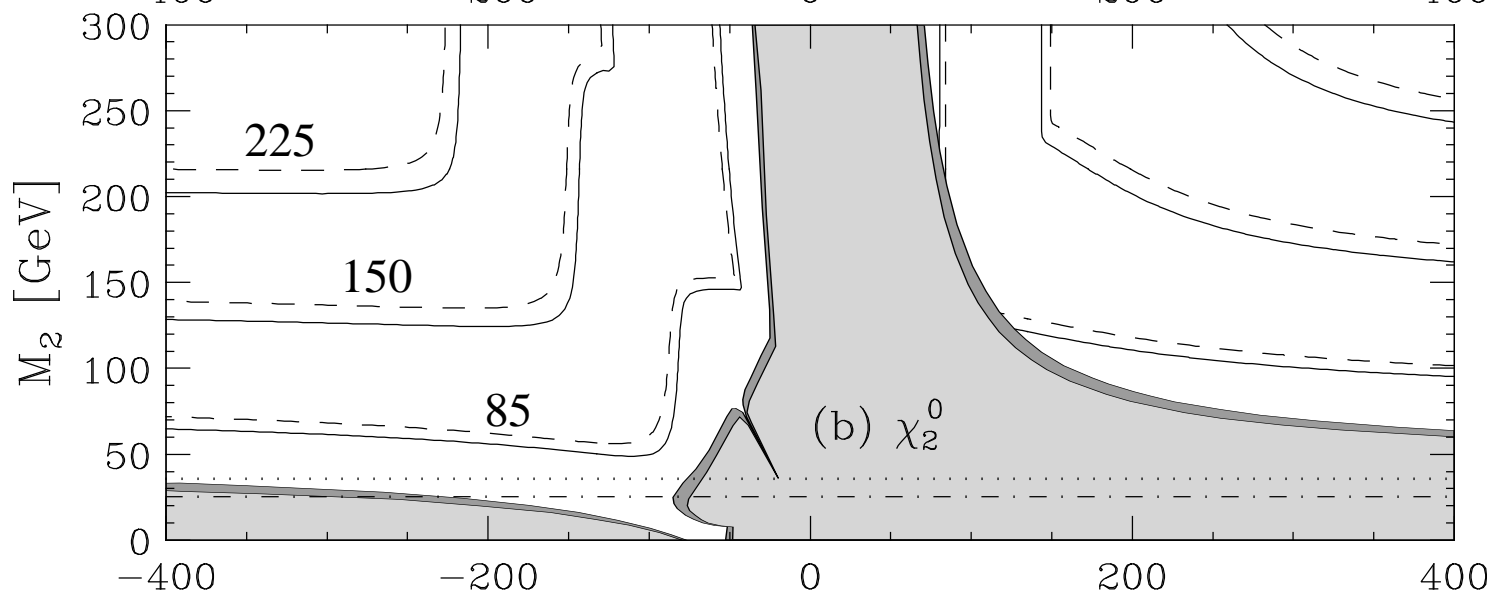
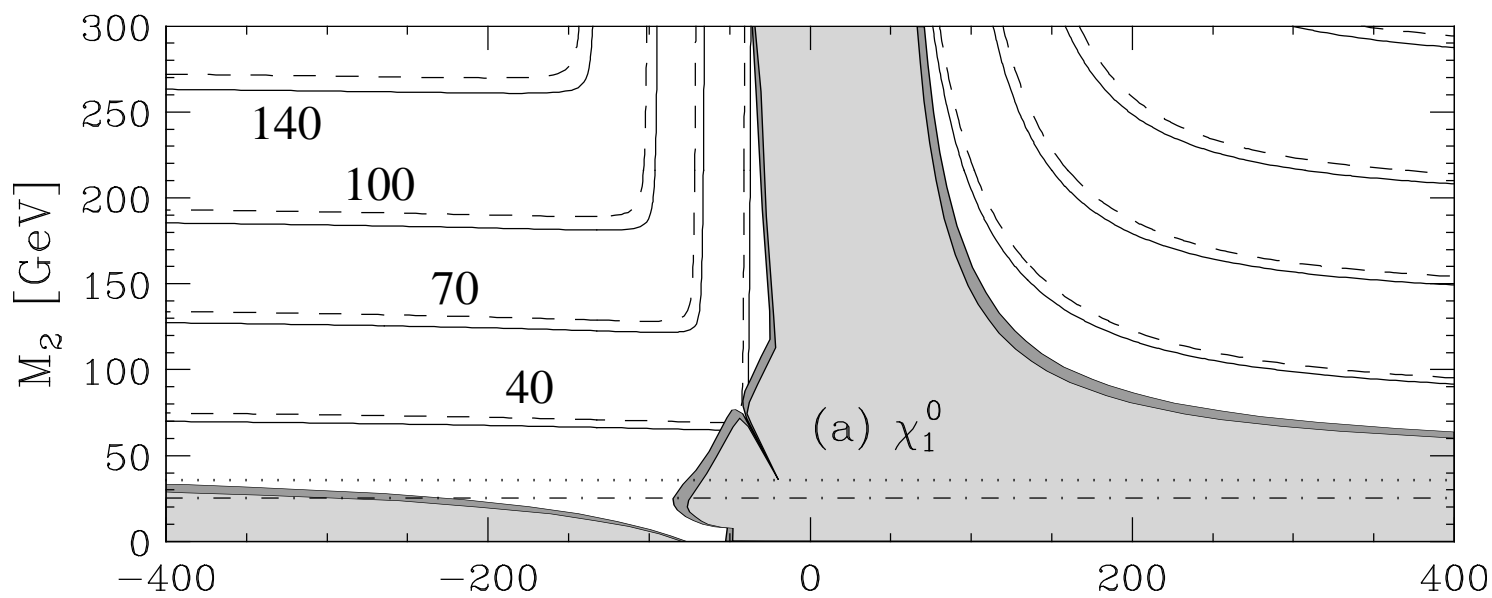
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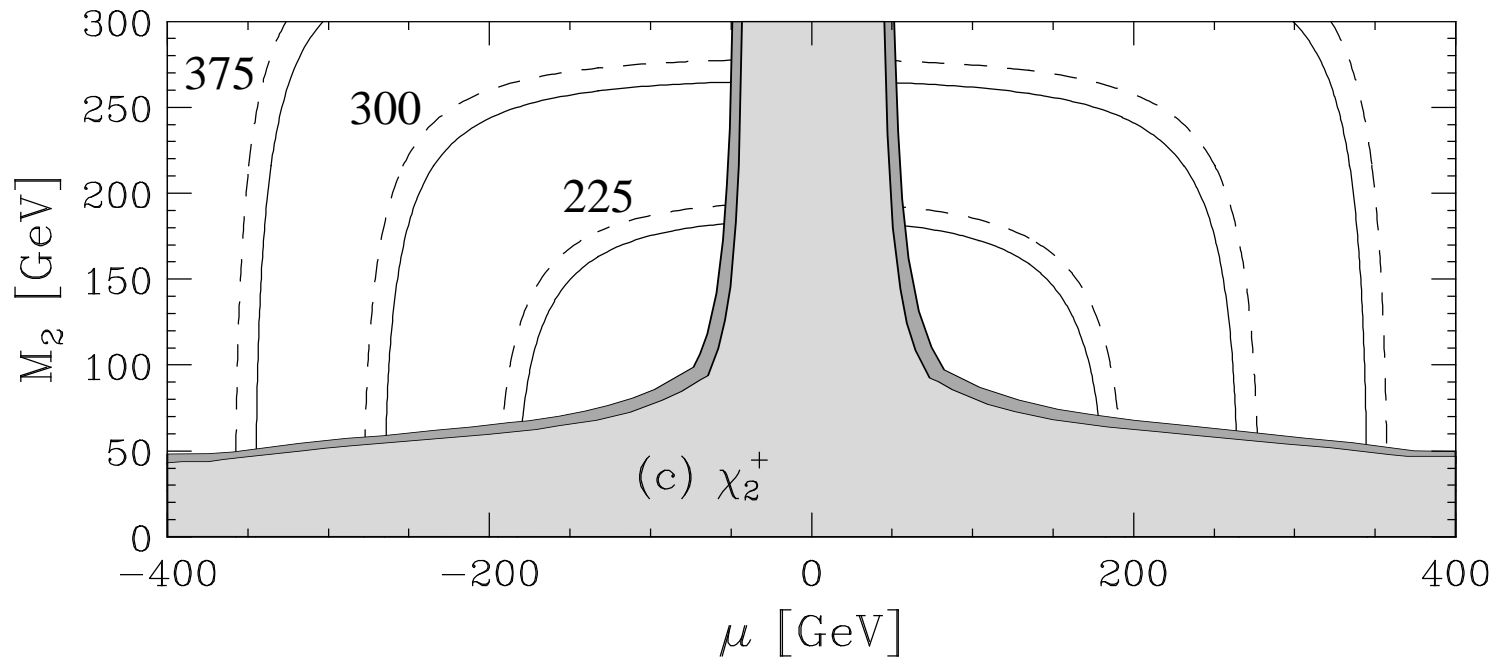
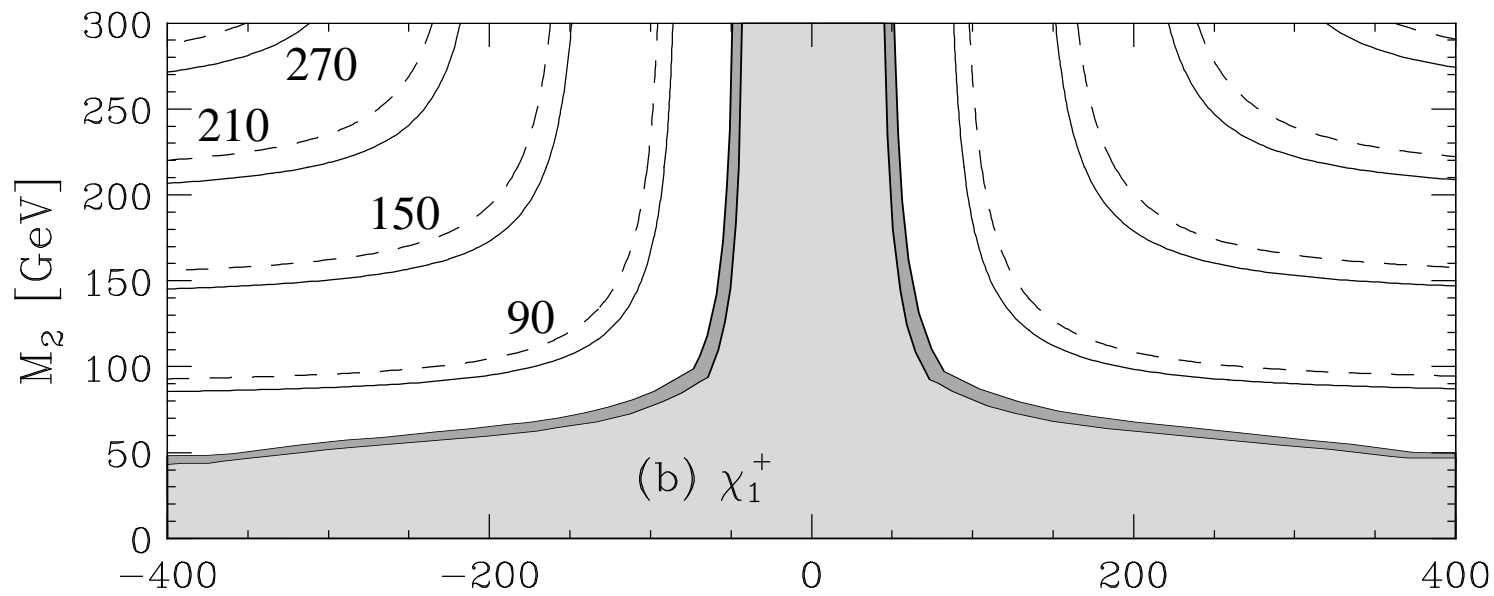
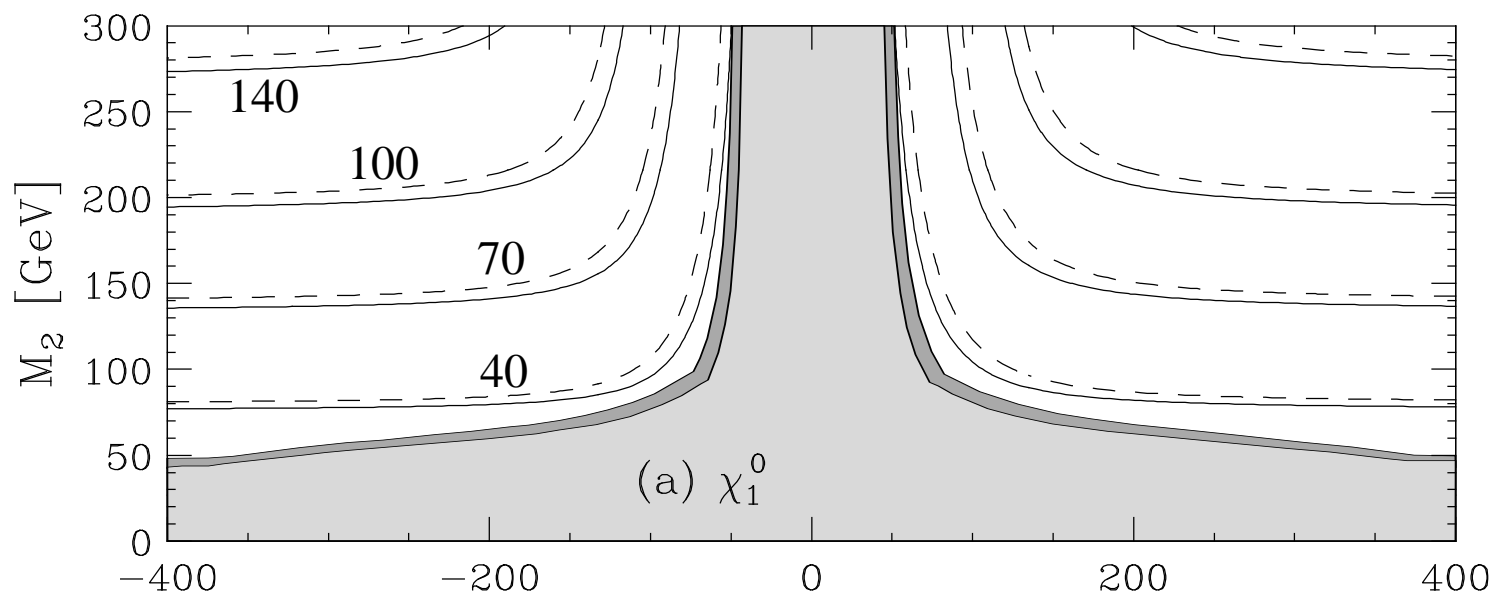




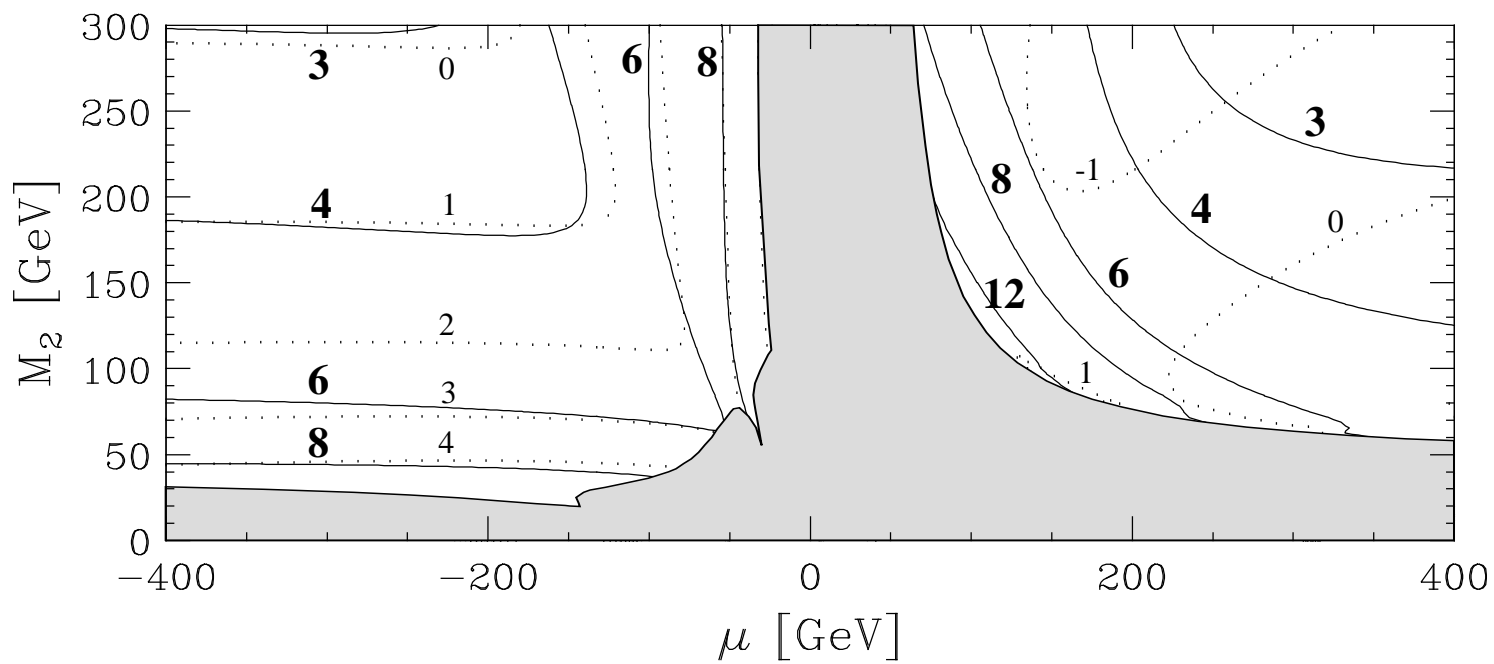
M_χ [GeV]; $\tan \beta = \sqrt{2}$



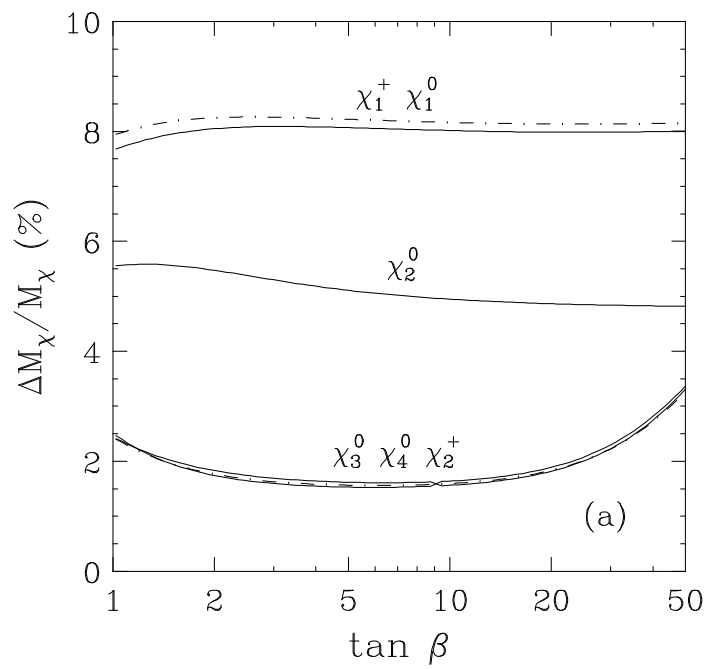
M_χ [GeV]; $\tan \beta=40$



$$\Delta M_{\chi_1^0}/M_{\chi_1^0} \text{ (\%)}; \tan \beta=2$$



$\mu = -400 \text{ GeV}, M_2 = 100 \text{ GeV}$



$\mu = -100 \text{ GeV}, M_2 = 400 \text{ GeV}$

